Semester One Examination, 2020

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 1

Section Two: Calculator-assumed

SOLUTIONS

WA student number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable): | |

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	15	15	100	98	65
				Total	100

Instructions to candidates

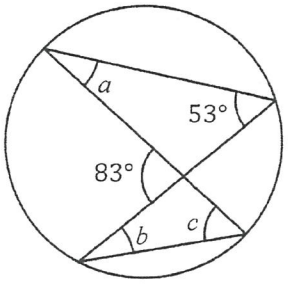
1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.
   Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

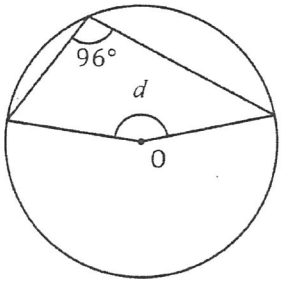
Markers use only		
Question	Maximum	Mark
9	6	
10	4	
11	9	
12	8	
13	7	
14	8	
15	3	
16	6	
17	4	
18	5	
19	8	
20	8	
21	8	
22	9	
23	5	
S2 Total	98	
S2 Wt (×0.6633)	65%	

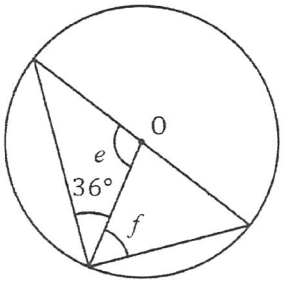
See next page

Question ~~10~~ 9 **(6 marks)**

Determine the size of the angles marked a, b, c, d, e and f shown in the circles below. Where marked, O is the centre of the circle.







$a = 30°$ ✓

$b = 30°$ ✓

$c = 53°$ ✓

$d = 168°$ ✓

$e = 108°$ ✓

$f = 54°$ ✓

Question ~~11~~ 10 **(4 marks)**

A coastguard boat is heading in the same direction as a fishing vessel at a velocity of $24\underset{\sim}{\mathbf{i}} - 10\underset{\sim}{\mathbf{j}}$ km/hr. If the coast guard is gaining on the fishing boat by 15 km/hr find the velocity of the fishing vessel.

$\sqrt{24^2+10^2} = 26$ km/hr ✓

$26 - 15 = 11$ km/hr ✓

$11 \times \left(\frac{24}{26}\underset{\sim}{i} - \frac{10}{26}\underset{\sim}{j}\right)$ ✓

$= \frac{132}{13}\underset{\sim}{i} - \frac{55}{13}\underset{\sim}{j}$ ✓

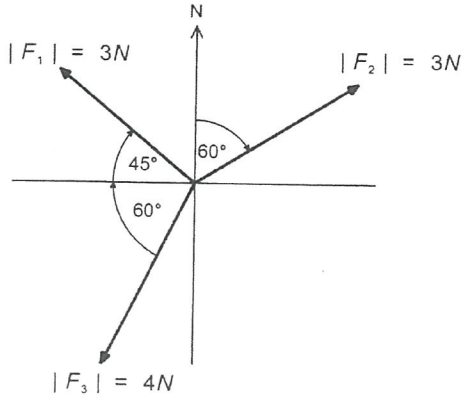
DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

See next page

Question ~~9~~ 11 9/~~18~~ **marks)**

A Physics teacher has a three way tug–of–war rope.
He is experimenting with his class.

The first experiment involves forces being applied to the rope as shown in Diagram 1.



(a) Determine the direction, correct to the nearest degree, of the resultant force experienced by the origin of the system. (5 marks)

$F_1 = \frac{-3\sqrt{2}}{2}\underset{\sim}{i} + \frac{3\sqrt{2}}{2}\underset{\sim}{j}$ ✓

$F_2 = \frac{3\sqrt{3}}{2}\underset{\sim}{i} + \frac{3}{2}\underset{\sim}{j}$ ✓

$F_3 = -2\underset{\sim}{i} - 2\sqrt{3}\underset{\sim}{j}$ ✓

$F_1 + F_2 + F_3 \sim -1.523\underset{\sim}{i} + 0.157\underset{\sim}{j}$ ✓

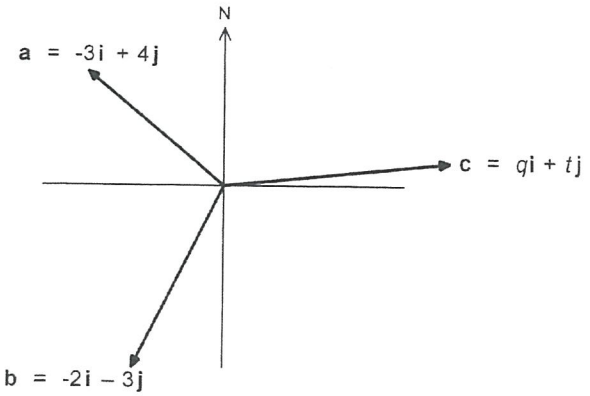
$\tan^{-1}\left(\frac{0.157}{1.523}\right) = 6^\circ$

$\therefore 276^\circ$ ✓

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

See next page

The second experiment involves keeping the rope in equilibrium using the forces as shown.



(b) Determine the magnitude and direction, to the nearest degree, of **c**. (4 marks)

$\begin{pmatrix}-3\\4\end{pmatrix} + \begin{pmatrix}-2\\-3\end{pmatrix} + \begin{pmatrix}q\\t\end{pmatrix} = 0$ ✓

$q = 5$
$t = -1$ } ✓

$|c| = \sqrt{5^2+1^2}$
$= \sqrt{26}$ ✓

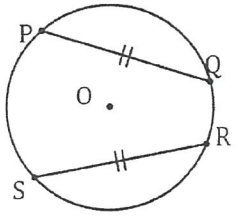
$\tan^{-1}\left(\frac{1}{5}\right)$
$= 11.3^\circ$

$\therefore 101.3^\circ$ + ✓

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 12 **(8 marks)**

(a) Prove that chords of equal length subtend equal angles at the centre of a circle. (3 marks)



Solution
$PQ = RS$ (given) $OP = OQ = OR = OS = r$ (all radii) Hence $\Delta OPQ \equiv \Delta ORS$ (SSS) Hence $\angle POQ = \angle ROS$ - chords of equal length subtend equal angles at the centre.
Specific behaviours
✓ establishes congruency of sides ✓ establishes congruency of triangles ✓ concludes equal angles

(b) Points A and B lie on a circle of radius 9.7 cm so that $AB = 13$ cm. Determine

(i) the distance of chord AB from the centre of the circle. (3 marks)

Solution
Let midpoint of chord be M. Then $OM^2 = r^2 - AM^2$ $OM = \sqrt{9.7^2 - 6.5^2}$ $= 7.2$ cm
Specific behaviours
✓ uses/defines midpoint or sketch diagram ✓ indicates correct method ✓ correct distance

(ii) the angle subtended by chord AB at the centre of the circle. (2 marks)

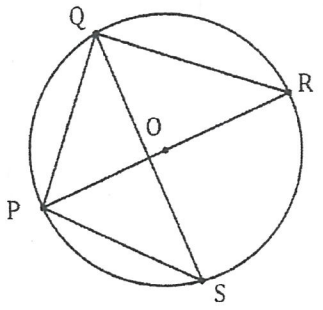
Solution
Let $\theta = \angle AOM$ (half angle required). Then $\theta = \sin^{-1}\left(\frac{11.5}{27.7}\right)$ $= 42.08°$ $\angle AOB = 2\theta$ $\approx 84.2°$
Specific behaviours
✓ indicates correct method ✓ correct angle

See next page

Question 13 **(7 marks)**

(a) The diagram shows points P, Q, R and S that lie on the circumference of a circle centre O. PR is a diameter and the size of $\angle PSQ = 53°$.

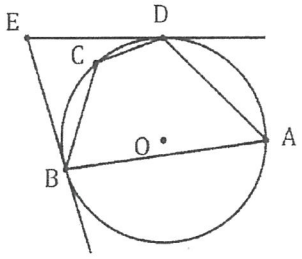
Determine, with reasons, the size of $\angle QPR$. (3 marks)



Solution
$\angle PRQ = \angle PSQ = 53°$ (angles on same arc) $\angle PQR = 90°$ (angle in semicircle) $\angle QPR = 180° - 90° - 53° = 37°$ (angle sum in triangle)
Specific behaviours
✓ uses angles on same arc ✓ uses angle in semicircle ✓ correct angle, using angle sum in triangle

(b) In the diagram shown, A, B, C and D are points on the circumference of a circle with centre O. Tangents to the circle at B and D intersect at E.

Determine, with justification, the size of $\angle BED$ when $\angle BCD = 134°$. (4 marks)



Solution
$\angle BAD = 180° - 134° = 46°$ (opposite angles in cyclic quadrilateral) $\angle BOD = 2(46°) = 92°$ (centre-circumference angles) $\angle OBE = \angle ODE = 90°$ (radius-tangent angle) $\angle BED = 360° - 180° - 92° = 88°$ (angle sum of quadrilateral $BODE$)
Specific behaviours
✓ uses opposite angles in cyclic quadrilateral ✓ uses angle at centre-circumference ✓ uses radius-tangent angle ✓ correct $\angle BED$

Question 14 **(8 marks)**

(a) An art gallery plans to display 5 paintings in a row. Determine how many arrangements of paintings are possible if they have a selection of 12 different paintings to choose from. (2 marks)

Solution
${}^{12}P_5 = 12 \times 11 \times 10 \times 9 \times 8 = 95\,040$
Specific behaviours
✓ indicates method
✓ correct number of arrangements

(b) In another room, the gallery plan to hang 7 different paintings in a row. If 3 of the paintings are by the artist Tyler, determine the number of different arrangements of paintings that are possible when

(i) the paintings by Tyler must be next to each other. (2 marks)

Solution
$3! \times 5! = 720$
Specific behaviours
✓ groups Marr together
✓ correct number of arrangements

(ii) a painting by Tyler must be at each end. (2 marks)

Solution
$3 \times 2 \times 5! = 720$
Specific behaviours
✓ uses 6!
✓ correct number of arrangements

(iii) the paintings by Tyler must be apart and not at the ends. (2 marks)

Solution
4 non-Tyler leave 3 spaces to hang Tyler in between (N_N_N_N): $n = 4! \times 3 \times 2 \times 1 = 144$
Specific behaviours
✓ indicates method
✓ correct number of arrangements

See next page

Question 15 **(3 marks)**

A calculator can generate random integers from 13 to 30. Use the pigeonhole principle to explain why 37 random integers should be generated to be certain that at least 3 of them are the same.

13 to 30 = 18 possible integers ✓

Let the 18 possible digits represent the pigeonholes

Worst case 2 in each pigeonhole

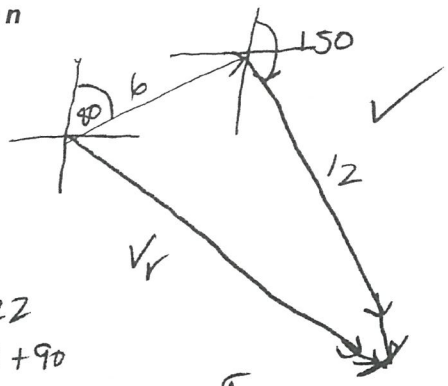
= 36 numbers ✓

∴ 37th number must be identical to two others ✓

Question 16 **(6 marks)**

Given ***m*** is a vector of magnitude 6 units and on a bearing of 080° and ***n*** is a vector of magnitude 12 units and on a bearing of 150°. Find the magnitude and direction of the following vectors.

(a) ***m*** + ***n*** (3 marks)

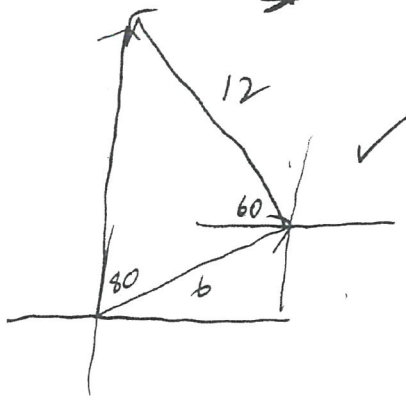


✓

15.14 units at 128° ✓ ✓

360 − 322
= 38 + 90

(b) −***n*** + ***m*** (3 marks)



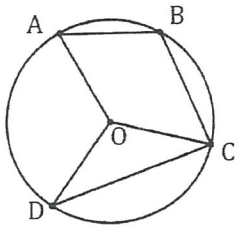
✓

11.43 units at 0° ✓ ✓

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question ~~18~~ 17 **(8 marks)**

(a) The vertices of quadrilateral $ABCD$ lie on the circumference of a circle centre O shown below. Given that $\angle ABC = 105°$ and $\angle ODC = 54°$, determine with reasoning the size of angle AOD. (4 marks)



Solution
$\angle DOC = 180° - 2\angle ODC$ (isosceles triangle) $\angle DOC = 180° - 2(54°) = 72°$ $\angle AOC = 2 \times \angle ABC = 210°$ (angles at centre-circumference) $\angle AOD = \angle AOC - \angle DOC = 210° - 72° = 138°$ (adjacent angles)

Specific behaviours
✓ uses isosceles triangles ✓ uses angles at centre-circumference ✓ uses adjacent angles ✓ correct angle

Question 18 **(5 marks)**

Three vectors are given by $\mathbf{a} = 5\mathbf{i} - 3\mathbf{j}$, $\mathbf{b} = -8\mathbf{i} + \mathbf{j}$ and $\mathbf{c} = x\mathbf{i} - 8\mathbf{j}$, where x is a constant.

Determine the value(s) of x if:

(a) **b** and **c** are perpendicular. (2 marks)

$-8x - 8 = 0$ ✓

$x = -1$ ✓

(b) the angle between **a** and **c** is 60°. (3 marks)

$\underset{\sim}{a} \cdot \underset{\sim}{c} = |a||c| \cos\theta$

$5x + 24 = \sqrt{34}\sqrt{x^2+64}\cos 60$ ✓

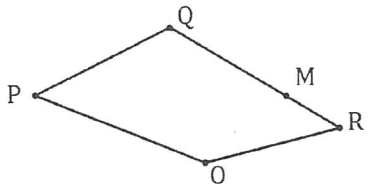
$(10x + 48)^2 = 34(x^2+64)$

$x = -14.41$ or -0.13 ✓

$\frac{240 \pm 136\sqrt{3}}{33}$

Question 19 **(8 marks)**

In quadrilateral $OPQR$ shown below, M lies on QR so that $|\overrightarrow{QM}| = 3|\overrightarrow{MR}|$.



(a) If $\overrightarrow{OP} = \mathbf{p}, \overrightarrow{OQ} = \mathbf{q}$ and $\overrightarrow{OR} = \mathbf{r}$, express the following in terms of $\mathbf{p}, \mathbf{q}$ and/or $\mathbf{r}$.

(i) $\overrightarrow{PR}$. (1 mark)

Solution
$\overrightarrow{PR} = \mathbf{r} - \mathbf{p}$
Specific behaviours
✓ correct expression

(ii) $\overrightarrow{RM}$. (2 marks)

Solution
$\overrightarrow{RM} = \frac{1}{4}\overrightarrow{RQ} = \frac{1}{4}(\mathbf{q} - \mathbf{r})$
Specific behaviours
✓ uses correct vector notation
✓ correct expression

(iii) $\overrightarrow{PM}$. (2 marks)

Solution
$\overrightarrow{PM} = \overrightarrow{PR} + \overrightarrow{RM}$ $= \mathbf{r} - \mathbf{p} + \frac{1}{4}(\mathbf{q} - \mathbf{r})$ $= \frac{3}{4}\mathbf{r} + \frac{1}{4}\mathbf{q} - \mathbf{p}$
Specific behaviours
✓ indicates suitable vector sum
✓ correct expression

(b) If O is the origin and points P, Q and R have coordinates $(-2, 39), (28, -14)$ and $(32, -18)$ respectively, determine the distance PM. (3 marks)

Solution
$\overrightarrow{PM} = \frac{3}{4}\begin{pmatrix} 32 \\ -18 \end{pmatrix} + \frac{1}{4}\begin{pmatrix} 28 \\ -14 \end{pmatrix} - \begin{pmatrix} -2 \\ 39 \end{pmatrix}$ $= \begin{pmatrix} 33 \\ -56 \end{pmatrix}$ $\left\|\begin{pmatrix} 33 \\ -56 \end{pmatrix}\right\| = 65$
Specific behaviours
✓ substitutes into expression for $\overrightarrow{PM}$
✓ $\overrightarrow{PM}$
✓ correct magnitude

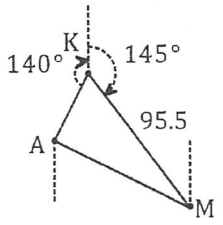
See next page

Question 20 **(8 marks)**

Oil platform K lies 95.5 km away from another oil platform M on a bearing of 325°. A steady current of 3.5 km per hour flows between the platforms on a bearing of 040°. A small boat at M, with a cruising speed of 15 km per hour, needs to arrive at K by 5 pm.

Determine the bearing that the boat should steer and the latest time it should depart from M.

Solution



$\angle K = 360° - 140° - 145° = 75°$

If journey takes t hours, then $MA = 15t$ and $AK = 3.5t$.

$\angle M$ using sine rule:

$$\frac{\sin M}{3.5t} = \frac{\sin 75°}{15t} \Rightarrow \sin M = \frac{3.5 \sin 75°}{15}$$

Hence $\angle M = 13.03°$ and $\angle A = 180° - 13.03° - 75° = 91.97°$.

Bearing to steer: $325° - 13.03° \approx 312°$

Distance AM using sine rule:

$$\frac{AM}{\sin 75°} = \frac{95.5}{\sin 91.97°} \Rightarrow AM = 92.3$$

$$t = 92.3 \div 15 = 6.153 \text{ h} = 6 \text{ h } 9 \text{ m}$$

Hence steer on bearing 312° and leave before 10:51 am.

Specific behaviours

- ✓ sketch diagram
- ✓ angle at K
- ✓ equation using sine rule
- ✓ solves angles in triangle
- ✓ solves for second side in triangle
- ✓ journey time in hours
- ✓ correct time to leave
- ✓ correct bearing

See next page

Question 21 **(8 marks)**

A school yearbook is produced by a committee of 2 teachers and 6 students. 5 teachers and 16 students have nominated for the committee.

(a) Determine how many different committees could be formed from the nominations. (2 marks)

Solution
$\binom{5}{2}\binom{16}{6} = 10 \times 8008 = 80\,080$
Specific behaviours
✓ chooses teachers and students separately
✓ correct number

(b) The student nominations include two sets of twins. Determine how many different committees could be chosen that do not include a set of twins. (4 marks)

Solution
Choose students with no set of twins (Set A, Set B, Others):
$n = \binom{2}{0}\binom{2}{0}\binom{12}{6} + \binom{2}{1}\binom{2}{1}\binom{12}{4} + \left[\binom{2}{1}\binom{2}{0} + \binom{2}{0}\binom{2}{1}\right]\binom{12}{5}$
$= 924 + 1980 + 3168$
$= 6072$
Ways to choose whole committee: $\binom{5}{2} \times 6072 = 60\,720$
Specific behaviours
✓ indicates isolation of cases
✓ uses systematic approach
✓ correct ways to choose students
✓ correct number of committees

(c) Suppose one of the students in the committee will be appointed as treasurer and another student will be appointed as secretary. Determine how many different committees can be formed with this structure. (2 marks)

Solution
Select a student and another, select other students and teachers:
$16 \times 15 \times \binom{14}{4}\binom{5}{2} = 16 \times 15 \times 10010$
$= 2\,402\,400$
Specific behaviours
✓ indicates correct method
✓ correct number

See next page

Question 13 22 **(9 marks)**

(a) A body is moving at 14 m/s on a bearing of 135°. Determine the equivalent velocity vector in exact form $a\mathbf{i} + b\mathbf{j}$. (2 marks)

Solution
$14\begin{pmatrix}\cos(-45)\\ \sin(-45)\end{pmatrix} = 7\sqrt{2}\mathbf{i} - 7\sqrt{2}\mathbf{j}$ m/s *NB Might choose to use CAS*
Specific behaviours
✓ indicates correct method ✓ correct vector using exact values

(b) Determine the bearing and speed of a body moving with velocity $-14\mathbf{i} - 5.1\mathbf{j}$ m/s. (2 marks)

Solution
Speed: $\sqrt{(-14)^2 + (-5.1)^2} = 14.9$ m/s Bearing: $180 + \tan^{-1}\left(\frac{14}{5.1}\right) = 180 + 70 = 250°$ *NB Might choose to use CAS*
Specific behaviours
✓ correct speed ✓ correct bearing

(c) The velocity vectors of particles P, Q and R are $\begin{pmatrix}-11\\ x\end{pmatrix}$, $\begin{pmatrix}y\\ 5\end{pmatrix}$ and $\begin{pmatrix}-5.6\\ 2\end{pmatrix}$ m/s respectively. If particles P and Q have the same speed and particles Q and R are moving in the same direction, determine the values of x and y. (5 marks)

Solution
$\begin{pmatrix}y\\ 5\end{pmatrix} = k\begin{pmatrix}-5.6\\ 2\end{pmatrix} \Rightarrow k = 2.5$ $y = 2.5 \times -5.6 = -14$ $\left\lvert\begin{pmatrix}-11\\ x\end{pmatrix}\right\rvert = \left\lvert\begin{pmatrix}-14\\ 5\end{pmatrix}\right\rvert$ $121 + x^2 = 196 + 25$ $x^2 - 100 = 0$ $x = \pm 10$
Specific behaviours
✓ uses parallel vectors to obtain k ✓ value of y ✓ equates magnitudes ✓ expands and simplifies ✓ both values for x

See next page

Question 23 **(5 marks)**

Solve for n, if ${}^{n}P_3 : {}^{n}C_5 = 1 : 5$

$$\frac{n!}{(n-3)!} : \frac{n!}{(n-5)!\,5!} \checkmark$$

$$A \times (n-1)(n-2) : \frac{A(n-1)(n-2)(n-3)(n-4)}{5!}$$

$$1 : \frac{(n-3)(n-4)}{5!} \checkmark$$

$$5! : n^2 - 7n + 12$$

$$120 : n^2 - 7n + 12$$

$$\frac{120}{n^2-7n+12} = \frac{1}{5} \checkmark$$

$$n^2 - 7n - 588 = 0$$

$$n = 28 \text{ or } n = -21 \checkmark$$

$$\therefore n = 28 \checkmark$$

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

End of questions